IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
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N06/5/MATSD/SP2/ENG/TZ0/XX/M+

MARKSCHEME

November 2006

MATHEMATICAL STUDIES

Standard Level

Paper 2

19 pages

Paper 2 Markscheme
Instructions to Examiners

Notes: **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

1 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

G Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, irrespective of working shown.

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently, marks not awarded.

ft Marks that can be awarded as **follow through** from previous results in the question

In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**. Marks to be awarded for unsupported answers are designated ***G*** in the mark scheme as such answers will usually arise from working performed on a graphic display calculator.

2 Method of Marking

(a) All marking must be done using a **red** pen.

(b) Marks must be noted on candidates' scripts as in the markscheme:

- Show the breakdown of individual marks using the **abbreviations** ***(M1)***, ***(A2)*** *etc*;
- Write down each part mark total, as indicated on the **question paper**. These totals should be written in the **margins** of the candidates' answer booklets;
- Write down and circle the total for each question at the end of the question.
- Transfer the total for **each question** to the front cover sheet and write down the total mark for the paper

(c) Working crossed out by the candidate should not be awarded any marks.

(d) Where candidates have written two solutions to a question, only the first solution should be marked.

(e) If correct working results in a correct answer but then further working is developed, full marks are **not** always awarded. In most such cases it will be a single final answer mark that is lost.

Full marks **can** be awarded if the candidate demonstrates clear understanding of the task and the result. If in doubt, consult your team leader.

(f) Candidate drawn graphs will have a single ***(A1)*** available for scales and labels combined. This can be awarded if all these are present and correct, even if no graph is drawn, however, the mark should not be awarded if the scale shown is inappropriate to, or inadequate for, the required missing graph. In papers which have two candidate drawn graphs, consistent errors in showing labels or scales can follow through on the second graph, though not if the error is complete omission of these features.

Please note: Assignment of marks to the answers in all the following examples is for demonstration purposes only. Marks for actual examination questions will not necessarily follow the same pattern.

Question: Using Pythagoras to find a side of a triangle:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ ***(M1)(A1)*** (3.61 3s.f.) **OR** Answer only ***(G2)***	Case (i) $\sqrt{13}$ or 3.61 or both Case (ii) $\sqrt{9+4} = \sqrt{13}$ = 6.50	***(G2)*** ***(M1)*** ***(A0)***

Question: Calculate the gradient of the line passing through the points (5,3) and (0,9).

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5} = -\frac{6}{5}$ ***(M1)(A1)*** **OR** Answer only ***(G1)***	(i) -6/5 (ii) $\frac{9-3}{0-5} = -\frac{6}{5}$ Gradient is -6/5 y = -6x/5 +9 (iii) $\frac{9-3}{0-5} = -\frac{6}{5}$ y = -6x/5 +9	***(G1)*** ***(M1)*** ***(A1)*** *(There is clear understanding of the gradient.)* ***(M1)*** ***(A0)*** *(There is confusion about what is required.)*

Question: sine rule used to find angle *A*, with angle *B* and side *b* known but side *a* is first calculated using Pythagoras in an adjoining triangle.

Markscheme			Candidate's Script		Marking
$a = \sqrt{25+36} = \sqrt{61}$		***(M1)(A1)***	Case (i)	$a = \sqrt{61}$	***(G2)***
OR	answer only	***(G2)***			
				$A = 55.9^\circ$	***(A2)***
$\frac{\sin(A)}{\sqrt{61}} = \frac{\sin(32)}{5}$		***(M1)(A1)***			
			Case (ii)	$A = 55.9^\circ$ *(with no mention of a)*	***(A2)***
$A = 55.9^\circ$		***(A1)***			
OR	answer only	***(A2)***			

3 Follow-through *(ft)* Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through *(ft)*** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '***ft***' appended to the eligible mark(s).

- If an answer resulting from follow through is extremely unrealistic (*e.g.* negative distances or wrong by a large order of magnitude) then the final ***A*** mark should not be awarded. If in doubt, contact your team leader.

- If a question is transformed by an error into a **different**, **much simpler question** then follow through might not apply or might be reduced. In this situation consult your team leader and record the decision on the candidate's script.

- To award follow through marks for a question part, **there must be working present for that part** and not just an answer based on the follow through. An isolated follow through answer, with no working, must be regarded as incorrect and receives no marks **even if it seems approximately correct**.

- Inadvertent use of radians will be penalised the first time it occurs. Subsequent use, even in later questions will normally be allowed follow through marks unless the answer is unrealistic. Cases of this kind will be addressed on an individual basis.

Implementation: The following examples illustrate correct use of the **follow through** process in straightforward situations.

Question: An investment problem with two different rates of interest and a total amount of \$600 split across the rates in consecutive periods:

Markscheme	Candidate's Script	Marking
(a) \$ 600 × 1.02 ***(M1)*** = \$ 612 ***(A1)*** **OR** answer only ***(G2)*** (b) $\$(\frac{612}{2} \times 1.02) + (\frac{612}{2} \times 1.04)$ ***(M1)*** = \$ 630.36 ***(A1)*****(ft)** **OR** answer only ***(G1)*** ***Note: The (M1) is for splitting the value from (a) and forming a sum of products.*** Here the **(ft)** indicates a possible follow through from part (a).	Case (i) (a) Final amount after 1st period = \$ 600 × 1.02 = \$ 602 (b) Amount after 2nd period = 301 × 1.02 + 301 × 1.04 = \$ 620.06 but note Case (ii) an ***(M0)*** almost always prohibits the **associated (ft)** so (a) \$ 600 × 1.02 = \$ 602 (b) \$ 602 × 1.04 = \$626.08 Case (iii) (a) \$ 600 × 1.02 = \$ 602 (b) No working. 620.06 given as answer. Case (iv) (a) \$ 612 (b) \$ 630.36	***(M1)*** ***(A0)*** ***(M1)*** ***(A1)*****(ft)** ***(M1)(A0)*** ***(M0)(A0)*****(ft)** ***(M1)(A0)*** ***(G0)*****(ft)** ***(G2)*** ***(G1)***

Question: Using trigonometry to calculate angles and sides of triangles.

Markscheme	Candidate's Script	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ ***(M1)(A1)***	(a) $\frac{\sin A}{4} = \frac{\sin 30}{3}$	***(M1) (A0) (use of sine rule but with wrong values)***
$A = 22.0^\circ$ ***(A1)***	$A = 41.8^\circ$	***(A0)***
OR answer only ***(A2)***		*(Note: the 2nd* ***(A1)*** *here was not marked* **(ft)** *and cannot be awarded, because there was an earlier error in the same question part.)*
(b) $x = 7 \tan A$ ***(M1)***	(b) case (i) $-x = 7 \tan A$	
$= 2.83$ ***(A1)*(ft)**	$= 6.26$	***(M1) (A1)*(ft)**
OR answer 2.83 only ***(G1)***	***but*** case (ii) 6.26	***(G0)***

4 Using the Markscheme

This markscheme presents a particular way in which each question might be worked and how it should be marked.

(a) As ***A*** marks are normally **dependent** on the preceding ***M*** mark being awarded, it is **not** possible to award ***(M0)(A1)***. Once an ***(M0)*** has been awarded, all subsequent ***A*** marks are lost in that part of the question, even if calculations are performed correctly, until the next ***M*** mark, unless otherwise instructed in the markscheme. (See the finance example above).

Similarly ***(A1)(R0)*** cannot be awarded for an answer which is accidentally correct for the wrong reasons given.

Example: Question: (a) χ^2 calculated followed by (b) degrees of freedom found and (c) and (d) comparison to critical value. (Dependence of ***A*** and ***R*** marks.)

Markscheme	Candidate's Script	Marking
(a) $\chi_{calc}^{\ 2} = 3.92$ ***(A1)***	Case (i)	
	(a) $\chi_{calc}^{\ 2} = 3.92$	***(A1)***
(b) $n = 4$ ***(A1)***	(b) $n = 4$	***(A1)***
(c) $\chi_{crit}^{\ 2} = 9.488$ ***(A1)*(ft)**	(c) Don't know?	***(A0)***
(d) Do not reject null hypothesis ***(A1)*(ft)**	(d) Do not reject null hypothesis	***(A0)*(ft)**
because $\chi_{calc}^{\ 2} < \chi_{crit}^{\ 2}$ ***(R1)*(ft)**	because $\chi_{calc}^{\ 2} > 0$	***(R0)*(ft)** *((A0) was awarded here because the reason is wrong.)*

	Case (ii) (a) $\chi_{calc}^2 = 3.92$ (b) $n = 4$ (c) $\chi_{crit}^2 = 4.488$ (d) Do not reject null hypothesis because $\chi_{calc}^2 < \chi_{crit}^2$ Case (iii) (a) $\chi_{calc}^2 = 3.92$ (b) n = 1 (c) $\chi_{crit}^2 = 3.841$ (d) Reject null hypothesis because $\chi_{calc}^2 > \chi_{crit}^2$	***(A1)*** ***(A1)*** ***(A0)*** ***(A1)*****(ft)** ***(R1)*****(ft)** ***(A1)*** ***(A0)*** ***(A1)*****(ft)** ***(A1)*****(ft)** ***(R1)*****(ft)**

(b) **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner that is consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR'** *etc*. This includes alternatives obtained with a graphic display calculator. In such cases, alternative ***G*** mark assignments for answer only will not be repeated if this is redundant.

Example: Question to find the coordinates of a vertex of a given quadratic.

Working	Marks
$f(x) = 2x^2 + 7x - 3$	
$x = -\frac{b}{2a} = -\frac{7}{4}$ ***(M1)*** *for use of – b/2a,* ***(A1)*** *for correct answer*	***(M1)(A1)*** **or** ***(G2)***
$f(-7/4) = -\frac{146}{16} = -\frac{73}{8}$ ***(M1)*** *for using f(-7/4),* ***(A1)*** *for answer.*	***(M1)(A1)*****(ft)** **or** ***(G1)***
Coordinates are (-7/4, -73/8)	***(A1)*****(ft)**
OR	**OR**
(-7/4,-73/8) (with no working at all)	***(G2)(G1)***

OR	**OR**
$f'(x) = 4x + 7$, $4x+7 = 0$	***(M1)***
so $x = -7/4$ ***(M1)*** *for attempting to take a derivative and setting it to 0* ***(A1)*** *for answer*	***(A1)***
$f(-7/4) = -\frac{146}{16} = -\frac{73}{8}$ ***(M1)*** *for using f(-7/4),* ***(A1)*** *for answer.*	***(M1)(A1)*****(ft)**
Coordinates are (-7/4,-73/8)	***(A1)*****(ft)**

(c) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

(d) As this is an international examination, all valid **alternative forms of notation** should be accepted.

Some examples of these are:

Decimal points: 1.7; 1'7; $1 \cdot 7$; 1,7.

Different descriptions of an interval: $3 < x < 5$; (3, 5);] 3, 5 [.

Different forms of notation for set properties (*e.g.* complement): A'; $\bar{A}$; A^c; $U - A$; (*A*

Different forms of logic notation: $\neg p$; p' ; $\tilde{p}$; $\bar{p}$; ~ *p*.
$p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

(e) Discretionary ***(d)*** marks: There will be rare occasions where the markscheme does not cover the work seen. In such cases, ***(d)*** should be used to indicate where an examiner has used discretion. It must be accompanied by a brief note to explain the decision made.

5 Accuracy of Answers

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to 3 significant figures.

A penalty known as an **ACCURACY PENALTY** ***(AP)*** is applied if an answer is either

(i) rounded incorrectly to 3 significant figures or

(ii) rounded correctly or incorrectly to some other level of accuracy.

This penalty is applied to the **final answer** of a question part only. It applies **also** when an exact answer is incorrectly rounded.

THE ACCURACY PENALTY IS APPLIED AT MOST ONCE PER PAPER! Subsequent accuracy errors can be **ignored** and full marks awarded if all else is correct.

An accuracy penalty must be recorded in proximity to the incorrect answer as ***(A0)(AP).***
Examiners must record the occurrence of an accuracy penalty by writing ***(AP)*** next to the relevant question total on the front of the cover sheet.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. This is **NOT** an accuracy penalty. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding ***(M0)***.

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. If the rounding is incorrect, an accuracy penalty should be applied as detailed above. Exact answers such as $\frac{1}{4}$ can be written as decimals to less than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential.

Ratios of π and answers taking the form of square roots of integers **(even if exact squares)** or any rational power of an integer (*e.g.* $\sqrt{13}, 2^{2/3}, \sqrt[4]{5}, \sqrt{9}$) may be accepted as exact answers. All other powers (*e.g*. of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

Answers **with no supporting working** which are written correct to more than 3 significant figures should be marked according to the scheme for correct answers with no working, but with an ***(AP)*** then applied. When this happens, **(A2)** or **(G2)** can be split if necessary (*e.g.* ***(A1)(A0)(AP)*** or ***(G1)(G0)(AP)***). **Unsupported** answers with less than 3 significant figures must be deemed incorrect even if they seem approximately correct.

An accuracy penalty should not be applied to an answer that is already incorrect for some other reason.

Special cases

Answers involving units of currency can be accepted correct to 3 significant figures or correct to the nearest currency unit (*e.g*. dollar) or correct to the nearest hundredth unit (*e.g.* cent). Allow all these cases to follow through to later question parts.

An answer taken directly from the IB chi-squared statistical table can be given and used to the same level of accuracy as appears in the table (3 decimal places) or correct to 3 significant figures.

For judging equivalence between 3 significant figures and use of minutes and seconds for angles, guidelines have been issued to paper setters. This problem will be dealt with on an individual basis as the need arises.

Examples: The Pythagoras example used before:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ ***(M1)(A1)*** (3.61 3s.f.) **OR** answer only ***(G2)***	(i) 3.6 or 4	***(G0)***
	(ii) 3.60555	***(G1)(G0)(AP)***
	(iii) $\sqrt{9+4} = \sqrt{13}$	***(M1)***
	= 3.6	***(A0)(AP)***
	(iv) $\sqrt{9+4} = \sqrt{13}$	***(M1)***
	= 3.60555	***(A0)(AP)***
	(v) $\sqrt{9+4} = \sqrt{13} = 3.60$	***(M1)(A0)(AP)***
	(vi) $\sqrt{9+4} = \sqrt{14} = 3.74$	***(M1)(A0)***

If the question specified *e.g.* correct to 4 decimal places for the answer, then there would be one extra mark available as follows:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ ***(M1)(A1)*** **OR** answer only ***(G2)*** *(Note: requires more than 4 d.p.)* = 3.6056 (4 d.p.) ***(A1)*(ft)** **OR** answer only ***(G2)*** **OR** answer 3.606 or 3.61 only ***(G1)***	(i) 3.605551 = 3.6056 (4 d.p.)	***(G2)(A1)***
	(ii) $\sqrt{9+4} = \sqrt{13}$	***(M1)(A1)***
	= 3.606	***(A0)***
	(iii) 3.60555	***(G2)(A0)***
	(iv) 3.6056	***(G2)***
	(v) $\sqrt{9+4} = \sqrt{14}$	***(M1)(A0)***
	= 3.7417	***(A1)*(ft)**
	(vi) $\sqrt{9-4} = \sqrt{5}$	***(M0)(A0)***
	= 2.2361	***(A1)*(ft)** *(Note: this is a special case, where the initial **(M0)** does not*

		*determine the final **(A0)** because the correction to **4dp** is an entirely new task.)*
	(vii) 3.606	***(G1)***

Premature Rounding

Accuracy errors in a final answer, which result from premature rounding earlier in the same question part, should not receive an accuracy penalty. There are two situations. If there is a mark available for a prematurely rounded answer and the rounding occurs at this stage, then the inappropriate rounding should be penalised with ***(A0)*** but the answer can then be allowed to follow through to the end of the question. If the first stage of the answer is correct but rounded further on, then it should be penalised at an appropriate place close to where it is rounded. Some discretion should be used to deny a ***(ft)*** mark if the rounding is very bad and the answer far from its required value.

Example: Question: sine rule used to find angle *A*, with angle *B* and side *b* known but side *a* is first calculated using Pythagoras in an adjoining triangle.

Markscheme		Candidate's Script		Marking
$a = \sqrt{25+36} = \sqrt{61}$	***(M1)(A1)***	(i)	$a = \sqrt{25+36} = \sqrt{61}$ = 7.8	***(M1)*** ***(A0)***
OR answer only	***(G2)***			
$\frac{\sin(A)}{\sqrt{61}} = \frac{\sin(32)}{5}$	***(M1)(A1)*(ft)**		$\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$	***(M1)(A1)*(ft)**
$A = 55.9^\circ$	***(A1)*(ft)**		$A = 55.8^\circ$	***(A1)*(ft)**
OR answer only	***(G2)***	(ii)	$a = \sqrt{25+36} = \sqrt{61}$	***(M1)(A1)***
			$\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$	***(M1)(A0)***
			$A = 55.8^\circ$	***(A1)*(ft)**
		(iii)	$a = \sqrt{25+36} = \sqrt{61}$	***(M1)(A1)***
			$\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$	***(M1)(A0)***
			$A = 55.9^\circ$	***(A0)(AP)*(ft)** *(even though this is the answer to the question, the rounded answer does not follow from the given working.)*
		(iv)	$a = \sqrt{25+36} = \sqrt{61}$	***(M1)(A1)***

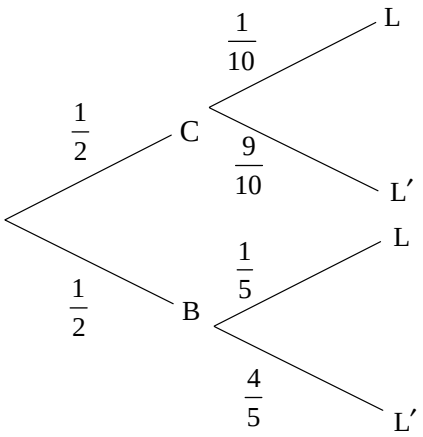
	$\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$	***(M1)(A0)***
	$A = \sin^{-1}(0.83) = 56.1^\circ$	***(A0)***
	(v) $a = \sqrt{25+36} = \sqrt{61} = 8$	***(M1)(A0)***
	$\frac{\sin(A)}{8} = \frac{\sin(32)}{5}$	***(M1)(A1)*(ft)**
	$A = 58.0^\circ$	***(A0)*(ft)** *(The rounding is severe and the answer quite far from correct).*
	(vi) $a = 7.8$	***(G0)***
	$A = 55.8^\circ$	***(G0)*(ft)** *(there is no working to justify the follow through.)*

6 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. In presenting their working they must use mathematical notation, not calculator notation. Correct answers supported only by calculator notation, without further explanation should be marked as answers without working and G marks awarded according to the mark scheme instructions.

Q1

(i) (a)



(A1) ***(A1)*** ***(A1)*** ***(A1)***

*Award **(A1)** for correct tree structure, **(A1)** for each complementary pair.*

[4 marks]

(b) (i) $\frac{1}{2}\times\frac{1}{10}$ ***(M1)***

$\frac{1}{20}$ (0.05, 5 %) ***(A1)(ft)(G2)***

*Award **(M1)** for correct product.*

[2 marks]

(ii) $\frac{1}{2}\times\frac{1}{10}+\frac{1}{2}\times\frac{1}{5}$ ***(M1)(M1)***

*Award **(M1)** for finding two products, **(M1)** for adding two products.*

$=\frac{3}{20}$ (0.15, 15 %) ***(A1)(ft)(G2)***

[3 marks]

(c) $\frac{\frac{1}{2}\times\frac{1}{5}}{\frac{3}{20}}$ ***(M1)***

*Award **(M1)** for using the conditional probability formula.*

$=\frac{2}{3}$, (0.667) ***(A1)(A1)(ft)***

or (G3)

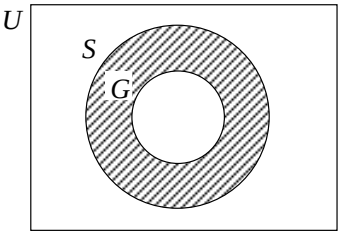
*Award **(A1)** for correct numerator, **(A1)** for correct denominator.*

[3 marks]

continued…

Q1 continued

(ii) (a)



(A1)(A1)(A1)

*Award **(A1)** for rectangle, **(A1)** for S drawn and named, **(A1)** for G completely inside S.*

[3 marks]

(b) shading on diagram ***(A1)*****(ft)**

[1 mark]

(c) sports cars that are not green ***(A2)***

*Award **(A1)** for sports cars intersecting with not green cars.*

[2 marks]

Total [18 marks]

Q2

(a) gradient $= \frac{4-0}{0-6}$ ***(M1)***

$= -\frac{2}{3}$ ***(A1) or (G2)***

[2 marks]

(b) midpoint $= \left(\frac{0+8}{2}, \frac{4+3}{2}\right)$

$= (4, 3.5)$ ***(A1)(A1)***

*Award **(A1)** if x and y coordinates not explicitly made clear.*

[2 marks]

(c) $AC = \sqrt{(0-8)^2 + (4-3)^2}$ ***(M1)***

*Award **(M1)** for using the distance formula and substituting the correct numbers.*

$= 8.06 \left(\sqrt{65}\right)$ ***(A1)(G2)***

[2 marks]

(d) Gradient BM $= \frac{3.5-0}{4-6}$ ***(M1)***

*Award **(M1)** for using values of B and M.*

$= -\frac{7}{4}$ ***(A1)*****(ft)*****(G2)***

$y = mx + c$

$0 = -\frac{7}{4} \times 6 + c$ ***(M1)***

*Award **(M1)** for using the equation of a straight line.*

$c = \frac{21}{2}$

$y = -\frac{7}{4}x + \frac{21}{2}$ *Can award **(G3)** for this with no working.* ***(A1)*****(ft)*****(G1)***

$7x + 4y - 42 = 0$ *This step can* **(ft)** *within part (d)* ***(A1)*****(ft)*****(G4)***

[5 marks]

(e) gradient AB $= -\frac{2}{3}$

gradient BC $= \frac{3}{2}$ ***(M1)***

*Award **(M1)** for attempting to find the gradient of BC.*

$-\frac{2}{3} \times \frac{3}{2} = -1$ ***(M1)***

*Award **(M1)** for multiplying their two gradients.*

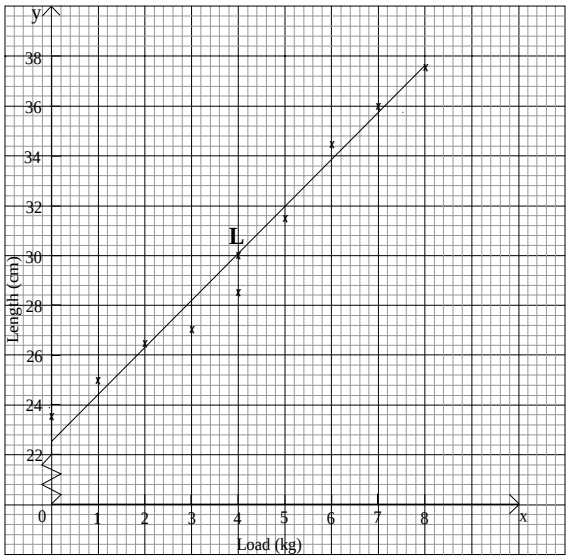
Yes, they are perpendicular. ***(A1)*****(ft)***

Accept any other valid mathematical method with working shown.

[3 marks]

Total [14 marks]

Q3 (a)



(A4)

*Award **(A1)** for correct scales and labels, **(A3)** for correct points, **(A2)** for 7 or 8 correct, **(A1)** for 5 or 6 correct.*

[4 marks]

(b) (i) 4 ***(G1)***

(ii) 2.58 ***(G1)***

(iii) 30 ***(G1)***

(iv) 4.78 ***(G1)***
If wrong version of s.d. used in (ii), can **(ft)** *in (iv) (5.07).*

[4 marks]

(c) L correctly plotted on graph and named ***(A1)*****(ft)**

[1 mark]

(d) (i) $\boldsymbol{r} = 0.986$ (0.987) ***(G1)***

(ii) (very) strong positive correlation ***(R1)*****(ft)*****(R1)*****(ft)**

[3 marks]

(e) $y = 1.83x + 22.7$ $(y = 1.825x + 22.7)$ ***(G1)(G1)***

*Award **(G1)** for $y = 1.83x$ $(1.825x)$, **(G1)** for* 22.7

[2 marks]

(f) Line drawn on graph. ***(A1)(A1)*****(ft)**
*Award **(A1)** for passing through the mean point, **(A1)** for y intercept between 22 and 23.*

[2 marks]

(g) (i) 32.6 cm
Allow margin of error of 0.2 from value on candidate's diagram. ***(A1)*****(ft)**

(ii) Not possible to find an answer as the value lies too far outside the given set of data. ***(R1)***

[2 marks]

Total [18 marks]

Q4 (i) (a) $(5k-2)-(2k+3)=(10k-15)-(5k-2)$ ***(M1)***

*Award **(M1)** for equating the common difference*

$5k-2-2k-3=10k-15-5k+2$

$3k-5=5k-13$ ***(A1)***

$-2k=-8$ or $2k=8$ ***(A1)***

$k=4$ ***(AG)***

OR **OR**

$(2k+3+10k-15)\div 2=5k-2$ ***(M1)(A1)***

*Award **(M1)(A1)** for finding the arithmetic mean with correct substitutions*

$2k+3+10k-15=10k-4$ ***(A1)***

$k=4$ ***(AG)***

[3 marks]

(b) 11, 18, 25 ***(A1)***

[1 mark]

(c) 7 ***(A1)***

[1 mark]

(d) $U_{20}=11+19\times 7$ ***(M1)***

*Award **(M1)** for using the correct formula with candidate's values*

$=144$ ***(A1)*****(ft)*****(G2)***

[2 marks]

(e) $S_{15}=\frac{15}{2}(2\times 11+14\times 7)$ ***(M1)***

$=900$ ***(A1)*****(ft)*****(G2)***

*Award **(M1)** for using the correct formula with candidate's values*

[2 marks]

(ii) (a) $XO^2=25^2-7^2$ ***(M1)***

*Award **(M1)** for using Pythagoras Theorem with correct signs and values*

$XO=24$ ***(A1)(G2)***

[2 marks]

(b) $\cos\alpha=\frac{7}{25}$ ***(M1)***

*Award **(M1)** for using any correct ratio.*

$\alpha=73.7^\circ$ ($73^\circ\ 44'$) ***(A1)*****(ft)*****(G2)***

Do not accept radians.

[2 marks]

(c) 120° ***(A1)***

[1 mark]

(d) $AB^2=7^2+7^7-2\times 7\times 7\times\cos 120^\circ$ ***(M1)(A1)***

*Award **(M1)** for using cosine rule.*

$AB=12.1\ (\sqrt{147})$ ***(A1)*****(ft)*****(G2)***

*Award **(M1)** for substituting values from the problem into the cosine rule, **(A1)** for correct values.*

Accept alternative, correct methods

[3 marks]

continued…

Q4 continued

(e) $\cos\theta = \dfrac{25^2 + 25^2 - 147}{2\times 25\times 25}$ ***(M1)(A1)***

*Award **(M1)** for substituting values from the problem into the cosine rule, **(A1)** for correct values.*

$\theta = 28.1^\circ\ (28.0^\circ)$ ***(A1)*****(ft)*****(G2)***

*Accept 28°. If using an isosceles triangle, award **(M1)** for angle, **(A1)** for answer, **(A1)** for doubling.*

[3 marks]

Total [20 marks]

Q5 (a) $y = x + z + z$ ***(M1)***

*Award **(M1)** for writing a sensible equation.*

$xz = 162$

$z = \dfrac{162}{x}$ ***(M1)***

$y = x + \dfrac{2\times 162}{x}$ ***(M1)***

$y = x + \dfrac{324}{x}$ ***(AG)***

[3 marks]

(b) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 - \dfrac{324}{x^2}$ ***(A1)(A1)(A1)***

*Award **(A1)** for 1 and no other constant present, **(A1)** for –324, **(A1)** for $\dfrac{1}{x^2}$ or x^{-2}.*

[3 marks]

(c) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$

$1 - \dfrac{324}{x^2} = 0$ ***(M1)***

*Award **(M1)** for putting candidate's derivative equal to zero.*

$x^2 = 324$ ***(A1)*****(ft)**

$x = 18$ ***(A1)*****(ft)*****(G3)***

[3 marks]

(d) $y = 18 + 9 + 9$ ***(M1)***

*Award **(M1)** for adding three sides of rectangle.*

$= 36$ ***(A1)*****(ft)*****(G2)***

OR

$18 + \dfrac{324}{18}$ ***(M1)***

$= 36$ ***(A1)*****(ft)***

[2 marks]

(e) $a = 36$ ***(A1)***

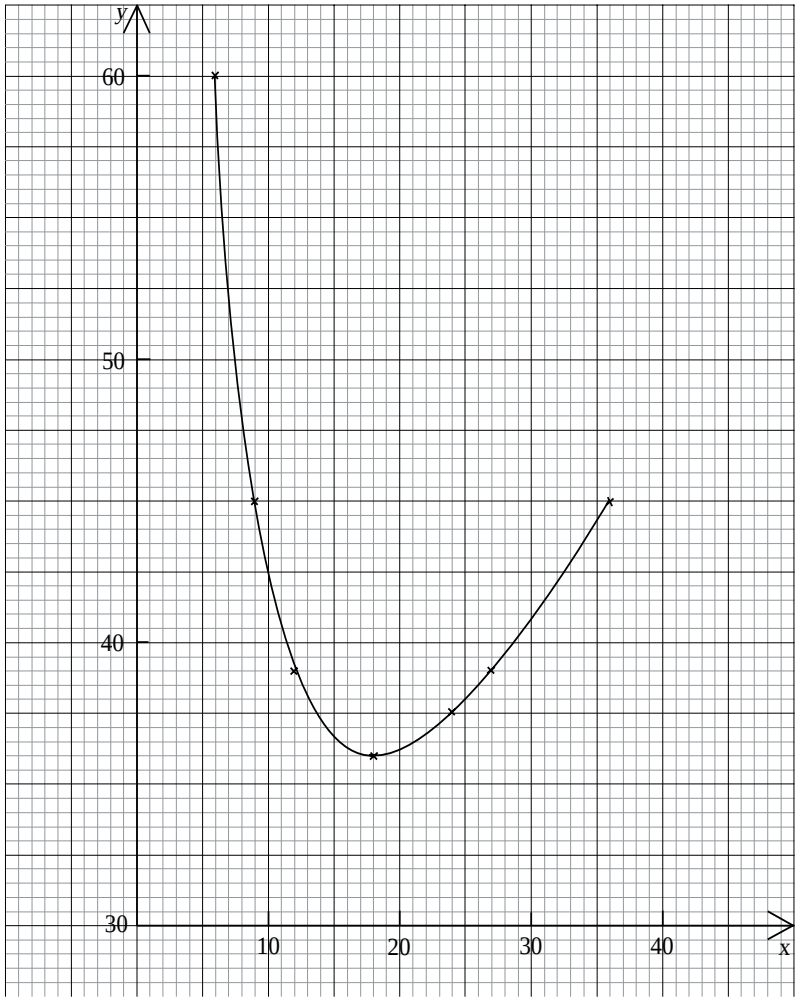
$b = 39$ ***(A1)***

[2 marks]

continued…

Q5 continued

(f)



***(A5)*(ft)**

*Award **(A1)** for correct scales and labels, **(A3)** for correct points plotted, **(A1)** for smooth curve with (18, 36) as the minimum value.*
*Award **(A2)** for 5 or 6 points correctly plotted, **(A1)** for 3 or 4 points correctly plotted.*

[5 marks]

(g) $x \geq 18$ ***(A1)(A1)*(ft)**

*Award **(A1)** for $x \geq$, **(A1)** for 18.*
Accept $x > 18$

[2 marks]

Total [20 marks]